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MODELING AND STUDY OF PIEZOELECTRIC POWER GENERATORS

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ABSTRACT

In the present study, a type of power generators of cantilever beam structure has been studied. A dynamic model describing the power generator, which takes into consideration the vibration of the mechanical structure, the power harvesting circuit, and the coupling between them, is proposed. A set of partial differential equations describing the dynamic model are obtained. Stability analysis is then carried out to analyze elementary properties of the generator. By using modal decomposition and state space representation, the set of partial differential equations could be easily solved no matter what the external excitation is, harmonic or non-harmonic, transient or continuous, at or off resonant frequency. The characteristics of the generators are analyzed and some recommendations on design of efficient power generators are provided.

KEYWORDS: piezoelectric, power harvesting, cantilever beam, modal decomposition, stability

1 INTRODUCTION

Power generators, which harvest mechanical energy from ambient sources to generate useful electric energy, are environmentally friendly products. With power generators, there is no need of worrying about replacing the batteries of remote monitors or portable personal electronics. Simply with vibration structure, power harvesting circuit, and some bridging component, the power generators then can get to work.

With all the advantages, the power generators attract a lot of attention among researchers[1-8]. Some applications have been proposed. For example, they can harvest power when attached to a bridge which vibrates under moving loadings. Or they can be installed in cell phones to harvest power while the carriers walk.

In the meanwhile, various modeling approaches have been proposed to elucidate the mechanisms of the power generators. However, a few drawbacks exist among them. For example, by assuming the vibration structure is of single degree of freedom[5,6], the structure has to vibrate around one of its natural frequencies (which is not necessarily possible), and the influence of other modes and the location of the piezoelectric patch is missing. Also, they intend to neglect the influence from the circuit to the vibration[7] although as shown in this work it is not negligible. Ajitsaria et al.[8] use an electric equivalent circuit to represent the vibration where some equivalent parameters have to be assumed or empirically obtained in formulating the model. In this work, we propose a model which successfully avoids the aforementioned drawbacks by using the vibration equation, circuit equation and the constitutive equation of the piezoelectric materials all together.

The paper is arranged as follows. The analytical model is first derived. Based on it, stability analysis is then carried out since stability analysis is a necessity in ascertaining whether the system designed can operate properly. From stability analysis, some elementary design guidelines are obtained by determining the stability domain of the initial points and parameters of the system. Subsequently, the analytical work is verified by numerical simulation. Finally, conclusions are drawn.

2 DEVELOPMENT OF THE MODEL

As shown in Fig. 1, a cantilever beam, called base, is adopted as the vibration structure of the power generator. A pair of piezoelectric patches (PZT patches) is bonded on the upper and lower surfaces of the base. The electrodes are placed on the upper and lower surfaces of each PZT patch and are insulated from the base material. The power harvesting circuit is represented as the PZT patches connecting with a resistor. Perfect bonding is assumed for all interfaces.

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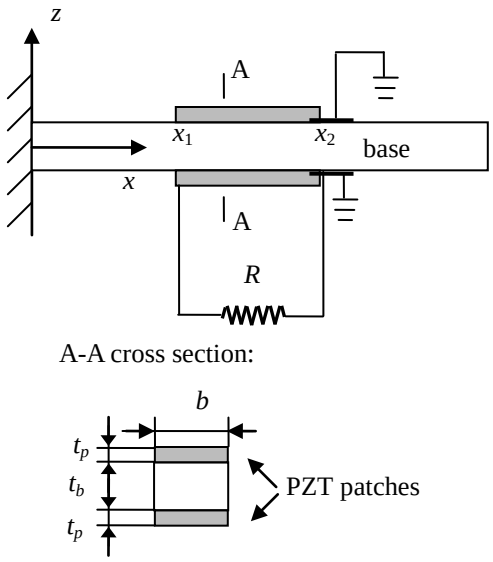

Figure 1 A schematic diagram of the generator. x1 and x2 are the locations of the left and the right edges of the PZT patches.

The Euler-Bernoulli beam theory is applied. The effect of the PZT patches is modeled as a distributed moment in the following flexural vibration equation

$$YI\frac{\partial^4 w}{\partial x^4}+\rho A\frac{\partial^2 w}{\partial t^2}=p-\frac{\partial m}{\partial x} \tag{1}$$

The moment from the PZT patches to the base per unit length, $m(x)$, is determined based on the geometry of the deformed shape:

$$m(x)=-kV\left[\delta(x-x_1)-\delta(x-x_2)\right] \tag{2}$$

where V is the voltage, $k=\dfrac{3Y_pY_bI_bd_{31}\left[\left(\frac{t_b}{2}+t_p\right)^2-\left(\frac{t_b}{2}\right)^2\right]}{2t_p\left\{Y_p\left[\left(\frac{t_b}{2}+t_p\right)^3-\left(\frac{t_b}{2}\right)^3\right]+Y_b\left(\frac{t_b}{2}\right)^3\right\}}$ is the coupling coefficient, d31 is the piezoelectric strain constant. Subscript b and p stand for the base and PZT patch, respectively.

Substituting Eq. (2) into Eq. (1), the vibration equation with electromechanical coupling effect is derived as

$$YI\frac{\partial^4 w}{\partial x^4}+\rho A\frac{\partial^2 w}{\partial t^2}=kV\left[\delta'(x-x_1)-\delta'(x-x_2)\right]+p(x,t) \tag{3}$$

To derive the circuit equation with electromechanical coupling effect, start with the constitutive equation of the piezoelectric material:

$$D_z=e_{31}\varepsilon_x+\kappa_{33}E_z \tag{4}$$

Integrating the component of the electric displacement normal to the electrode surface, the electric charge, Q, is obtained as

$$Q=\oint_{\tilde{A}}\vec{D}\cdot d\vec{A} \tag{5}$$

By substituting Eq. (4) into Eq. (5) and differentiating with respect to time, we obtain

$$\dot{Q}=-be_{31}\frac{t_b+t_p}{2}\int_{x_1}^{x_2}\frac{\partial^2\dot{w}}{\partial x^2}dx-b\kappa_{33}\frac{\dot{V}}{t_p}(x_2-x_1) \tag{6}$$

Multiplying the resistance, R, on both sides of Eq. (6) and considering $V=\dot{Q}R$, the electric circuit with electromechanical coupling effect is obtained as

$$V=-Rbe_{31}\frac{t_b+t_p}{2}\int_{x_1}^{x_2}\frac{\partial^2\dot{w}}{\partial x^2}dx-Rb\kappa_{33}\frac{\dot{V}}{t_p}(x_2-x_1) \tag{7}$$

By combining Eqs. (3) and (7), the analytical model for the power generator is obtained. Eqs. (3) and (7) are coupled differential equations where Eq. (3) is a partial differential equation. First, apply modal decomposition to transform them into ordinary differential equations. Assume that w can be expressed as a linear combination of the modal shape functions:

$$w(x,t)=\sum_{i=1}^{\infty}\eta_i(t)\Phi_i(x) \tag{8}$$

where η_i are the time dependant modal coordinates and Φ_i are the normalized orthogonal modal shape functions.

Substituting Eq. (8) into Eqs. (3) and (7) and using modal decomposition, the dynamic system can be reformulated as follows:

$$\ddot{\eta}_i+\omega_i^2\eta_i=\frac{kV}{\rho A}\left[\Phi_i'(x_2)-\Phi_i'(x_1)\right]+f_i(t),\quad i=1\cdots\infty \tag{9}$$

$$\frac{V}{R}+be_{31}\frac{t_b+t_p}{2}\sum_{i=1}^{\infty}\left[\Phi_i'(x_2)-\Phi_i'(x_1)\right]\dot{\eta}_i+\frac{b\kappa_{33}(x_2-x_1)}{t_p}\dot{V}=0 \tag{10}$$

By letting $\dot{\eta}_i=\xi_i$, $V=\zeta$, $k_1=\dfrac{k}{\rho A}$, $k_2=\dfrac{e_{31}\frac{t_b+t_p}{2}t_p}{\kappa_{33}(x_2-x_1)}$, $r_i=\Phi_i'(x_2)-\Phi_i'(x_1)$ and $\alpha=\dfrac{t_p}{b\kappa_{33}(x_2-x_1)R}$, the modal state space representation of the above equations is:

$$\begin{cases}\dot{\eta}_i=\xi_i\\ \dot{\xi}_i=-\omega_i^2\eta_i+k_1r_i\zeta+f_i(t)\quad i=1\cdots\infty\\ \dot{\zeta}=-k_2\sum_{i=1}^{\infty}r_i\xi_i-\alpha\zeta\end{cases} \tag{11}$$

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where the external excitation can be harmonic or non-harmonic, transient or continuous, at or off resonant frequencies.

3 LYAPUNOV STABILITY ANALYSIS

Lyapunov stability analysis is for checking the stability of a dynamic system. The direct method of Lyapunov involves searching a 'Lyapunov function' for a dynamic system. If such a function exists, then the system is stable. Before proceeding to the analysis, a few definitions and stability theorems are provided.

Positive Definite: A scalar function $\mathrm{V}(\mathbf{x})$ is said to be positive definite if (i) $\mathrm{V}(\mathbf{x}) \geq 0$ for all $\mathbf{x}$, (ii) $\mathrm{V}(\mathbf{x}) = 0$ if and only if $\mathbf{x} = \mathbf{0}$.

Negative Semi-Definite: A scalar function $\mathrm{V}(\mathbf{x})$ is said to be negative semi-definite if $\mathrm{V}(\mathbf{x}) \leq 0$ for all $\mathbf{x}$.

Theorem 1 [9]: For a dynamic system, if

(i) there exists a positive definite function $\mathrm{V}(\mathbf{x})$; and

(ii) its total derivative $\dot{\mathrm{V}}(\mathbf{x})$ along the solution of the system is negative semi-definite,

then the system is stable.

Theorem 2 [10,11]: For a dynamic system, if

(i) there exists a positive definite function $\mathrm{V}(\mathbf{x})$;

(ii) the function $\mathrm{V}(\mathbf{x})$ is radially unbounded, i.e. $\mathrm{V}(\mathbf{x}) \to \infty$ as $\mathbf{x} \to \infty$;

(iii) its total derivative $\dot{\mathrm{V}}(\mathbf{x})$ along the solution of the system is negative semi-definite; and

(iv) $\dot{\mathrm{V}}(\mathbf{x})$ does not vanish identically along any solution of the system except the null solution,

then the system is global asymptotically stable.

3.1 STABILITY OF THE EQUILIBRIUM POINT

Denote $\mathbf{x} = \left(\eta_1, \xi_1, \eta_2, \xi_2, \cdots, \zeta\right)^{\mathrm{T}}$. By letting $\dot{\mathbf{x}} = 0$ and $f_i(t) = 0$, from Equation (11), the equilibrium point of the system can be found to be $\mathbf{x} = 0$, i.e. the origin. In the following work, the direct Lyapunov method is applied to study the stability of the equilibrium point.

In order to construct a Lyapunov function, the sign of the coefficients in Equation (11) must be known. Since ω_i is the natural frequency, ω_i^2 is a positive number. k_1 and k_2 are both negative. α is a positive number. Based on the above knowledge, the Lyapunov function can be constructed as:

$$\mathrm{V}(\mathbf{x}) = \sum_{i=1}^{\infty} (\omega_i^2 \eta_i^2 + \xi_i^2) + \frac{k_1}{k_2} \zeta^2$$

Because $k_1/k_2>0$, $\mathrm{V}(\mathbf{x})$ is positive definite and radially unbounded. $\dot{\mathrm{V}}(\mathbf{x})$ along the solution of the system is:

$$\dot{\mathrm{V}}(\mathbf{x}) = 2\sum_{i=1}^{\infty} \left[\omega_i^2 \eta_i \dot{\eta}_i + \xi_i \dot{\xi}_i \right] + 2\frac{k_1}{k_2} \zeta \dot{\zeta} \qquad (12)$$

Substitute Equation (11) into Equation (12), we obtain:

$$\dot{\mathrm{V}}(\mathbf{x}) = 2\sum_{i=1}^{\infty} \left[\omega_i^2 \eta_i \xi_i + \xi_i \left(-\omega_i^2 \eta_i + k_1 r_i \zeta \right) + \frac{k_1}{k_2} \zeta \left(-k_2 r_i \xi_i \right) \right] - 2\frac{k_1}{k_2} \alpha \zeta^2 = -2\frac{k_1}{k_2} \alpha \zeta^2 \qquad (13)$$

$\dot{\mathrm{V}}(\mathbf{x}) \leq 0$ for all $\mathbf{x}$. Therefore, $\dot{\mathrm{V}}(\mathbf{x})$ is negative semi-definite. Based on Theorem 1, the dynamic system is stable.

Next to see if it is global asymptotically stable. From Equation (13), $\dot{\mathrm{V}}(\mathbf{x})=0$ means $\zeta = 0$. To ascertain whether $\dot{\mathrm{V}}(\mathbf{x})$ vanishes identically along any other solution of the system except the null solution, there is a need to check if the set of $M = \{\mathbf{x} : \dot{\mathrm{V}}(\mathbf{x}) = 0\}$ contains any solution other than the null one. First of all, calculate the total derivative of $\dot{\mathrm{V}}(\mathbf{x})$, i.e. $\dot{\zeta}$, along the solution of the system given by:

$$\dot{\zeta} = -k_2 \sum_{i=1}^{\infty} r_i \xi_i - \alpha \zeta \qquad (14)$$

Since $\mathbf{x}$ in the set M satisfies $\zeta = 0$, substitute $\zeta = 0$ into Equation (14):

$$\dot{\zeta} = -k_2 \sum_{i=1}^{\infty} r_i \xi_i \qquad (15)$$

Since ξ_i's are the modal functions of the structure, they are orthogonal. If all the r_i's are nonzero, all the ξ_i's have to be zero to achieve $\dot{\zeta} = 0$. Since all the ξ_i's are zero, all $\dot{\xi}_i$ satisfy $\dot{\xi}_i = 0$. From the equation in system (11) $\dot{\xi}_i = -\omega_i^2 \eta_i + k_1 r_i \zeta$, with $\dot{\xi}_i = 0$ and $\zeta = 0$, all η_i's have to be zero, too. In sum, when all the r_i's are nonzero, $\dot{\mathrm{V}}(\mathbf{x})$ vanishes only at the null solution of the system. From Theorem 2, the equilibrium point is global asymptotically stable.

If not all of the r_i's are nonzero, the dynamic system is stable but not global asymptotically stable.

3.2 STABILITY OF THE MOTION

In some circumstances, the generator may encounter continuous excitations. In this case, the dynamic system is described as Equation (11). In order to determine the stability of the undisturbed motion $\tilde{\mathbf{x}}$ of system (11), let

$$\mathbf{e} = \mathbf{x} - \tilde{\mathbf{x}} \qquad (16)$$

denote the difference between the disturbed motion and the undisturbed motion. Take time differential on both sides of Equation (16) with respect to time:

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$$\dot{\mathbf{e}} = \dot{\mathbf{x}} - \dot{\tilde{\mathbf{x}}} \tag{17}$$

By substituting Equation (11) into Equation (17), we obtain the following equation that governs the differences:

$$\begin{cases} \dot{e}_{\eta_i} = e_{\xi_i} \\ \dot{e}_{\xi_i} = -\omega_i^2 e_{\eta_i} + k_1 r_i e_\zeta \qquad i = 1 \cdots \infty \\ \dot{e}_\zeta = -k_2 \sum_{i=1}^{\infty} r_i e_{\xi_i} - \alpha e_\zeta \end{cases} \tag{18}$$

where $e_{\eta_i} = \eta_i - \tilde{\eta}_i$, $e_{\xi_i} = \xi_i - \tilde{\xi}_i$ and $e_\varsigma = \zeta - \tilde{\zeta}$. Following the procedure outlined in section 3.1, it can be proven that if all the r_i's are nonzero, system (18) is global asymptotically stable. Otherwise, system (18) is stable.

4 NUMERICAL SIMULATION

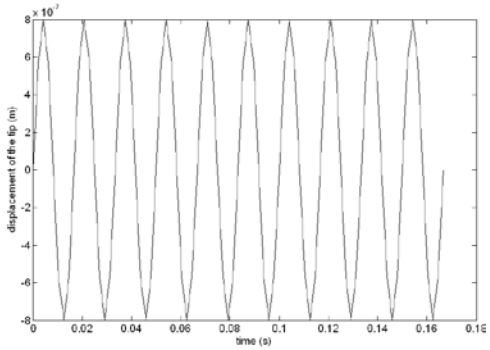
A numerical simulation, in which the base and piezoelectric materials are assumed to be silicon and PZT-5H, respectively, is carried out. The thickness and length of the beam are 250μm and 5mm, respectively. The width of the beam and the PZT patches is 0.5mm. Alter other parameters to see the effect on the power generating.

4.1 FREE VIBRATION CASE

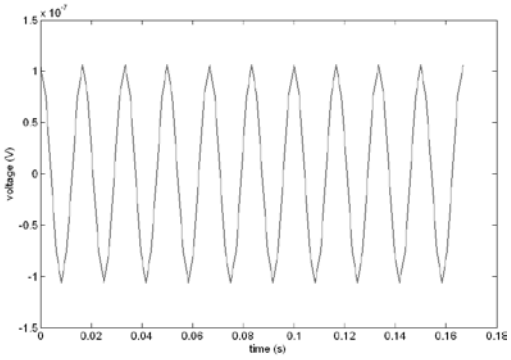
The external excitation is transient. The thickness of the PZT patches is 25μm. The electric resistance is 10 Ω. Alter the length and location of the PZT patches to see their effect.

(1) Location of the PZT patches: x_1 = 1mm， x_2 = 2mm

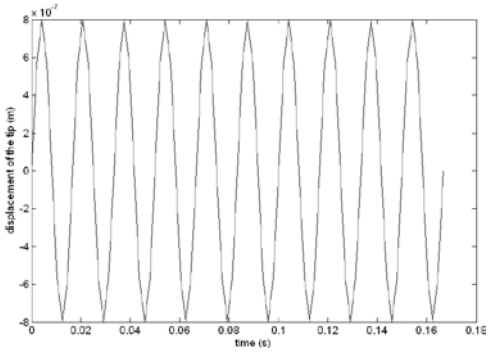
(2) Location of the PZT patches: x_1 = 0mm， x_2 = 2.3541mm



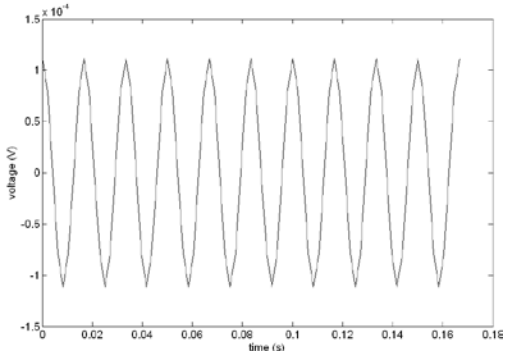
(a) displacement



(b) voltage



(c) displacement



(d) voltage

Figure 2 The evolution of the displacement at the tip (a), (c) and the generated voltage (b), (d).

After solving Eq. (11), the transverse displacement, $w(x,t)$, and the generated voltage, $V(t)$, are obtained and shown in Fig. 2, where Fig. 2(a) and 2(b) associate with location (1) and Fig. 2(c) and 2(d) for location (2).

For a system without damping, it will normally vibrate forever. However, with the electromechanical coupling, the system behaves as in Fig. 2(a), 2(b). That is as mechanical energy is gradually converted to electrical energy, the decrease of the former causes the decrease of the vibration amplitude, which further results in the decrease of voltage generated. A calculation shows that r_i of the first several important modes are non-zero indicating good coupling and energy conversion ability.

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Modify the PZT patches to location (2), which leads to r_2 = 0. The vibration amplitude decreases initially due to the conversion of mechanical energy to electrical energy associated with other modes than the second one. After this is done, the beam vibrates in its second mode shape forever when no mechanical damping is included. Therefore less power is harvested at location (2), which should be avoided in practical designs.

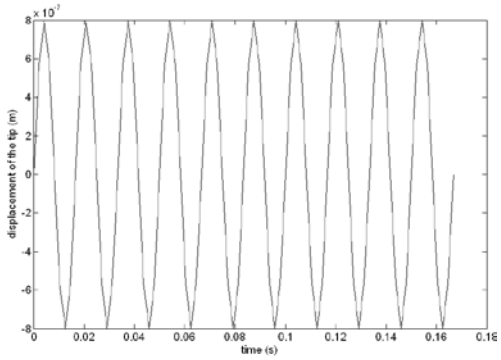
4.2 FORCED VIBRATION CASE

The external excitation is $p(x,t) = 1\text{N/m} \times \sin(\omega t)$, where ω = 377rad/s. The solutions correspond to two sets of parameters are obtained and plotted in Fig. 3, with Fig. 3(a), (b) for set (1) and Fig. 3(c), (d) for set (2).

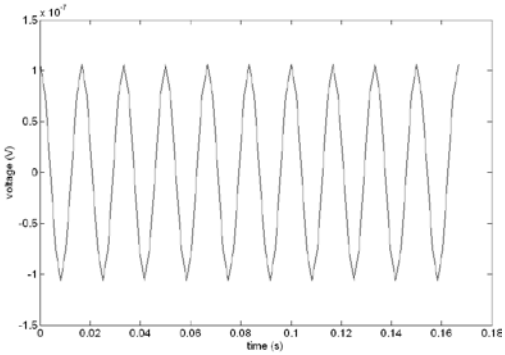
(1) Parameter set: x_1 = 1mm，x_2 = 2mm，t_p = 25μm,，R = 10Ω

(2) Parameter set: x_1 = 158.3μm，x_2 = 819.663μm，t_p = 12.957μm，R = 10000Ω

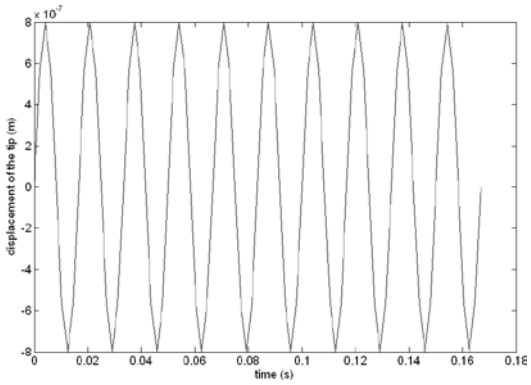
Comparing Fig. 3(a), (b) with Fig. 3(c), (d), under continuous external excitation, although mechanical energy keeps being converted to electrical energy, the vibration amplitude keeps unchanged due to the work done by the external excitation. However, the magnitude of the work done is different for the two parameter sets, which results in different levels of voltage generated. Therefore, by modifying the parameters, the power generated could be enhanced. This gives us a hint on how to improve the performance of the generator. It is carried out in another work..



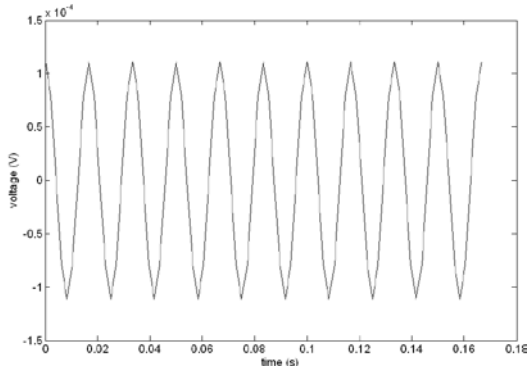
(a) displacement



(b) voltage



(c) displacement



(d) voltage

Figure 3 The evolution of the displacement at the tip (a), (c) and the generated voltage (b), (d).

5 CONCLUSION

In this work, a theoretical model describing the coupled system of the mechanical structure and the power harvesting electric circuit is proposed. The electromechanical coupling manifests its effect not only in the power harvesting circuit but also in the mechanical vibration in this model. By representing the system in the modal state space, one can easily determine the specific relation between the external excitation and the power that could be harvested. The characteristics of the generators are analyzed analytically and numerically and the parameters which affect the intensity of the electromechanical coupling and further the power generation are discovered. Some elementary design guidelines, e.g., the locations where r_i vanishes, namely, the slopes of the modal shape function at the two edges of the PZT patch equal, should be avoided in applying PZT patches, are provided.

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